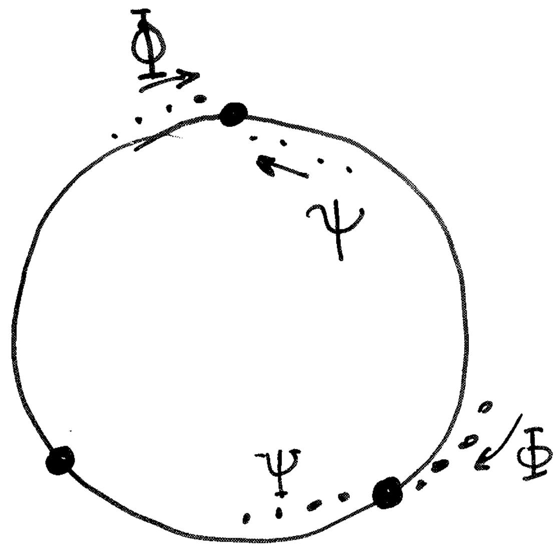
E. Forest

# The system




$$\begin{pmatrix}\Phi\\ \Psi\end{pmatrix}^{+} = \begin{pmatrix}A & A'\\ B & B'\end{pmatrix}\begin{pmatrix}\Phi\\ \Psi\end{pmatrix}^{-}$$

Invariant under the exchange of $\Psi \leftrightarrow \Phi$

$$\begin{pmatrix}\Phi\\ \Psi\end{pmatrix}^{+} = \left(I A + \Delta B\right)\begin{pmatrix}\Phi\\ \Psi\end{pmatrix}^{-}$$

$$\Delta = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

# Trick #1

Follow the site instead of bunch.

$$\Gamma = (\Phi, \Psi) = (\underbrace{\varphi^1, \psi^1}_{\gamma^1}, \cdots, \varphi^N, \psi^N)$$

$\Omega$ = relocation operator.

$$\Omega\Gamma = (\varphi^N, \psi^2, \varphi^1, \psi^3, \cdots, \varphi^{N-1}, \psi^1).$$

"$\frac{1}{N}$ of a turn"

$$\Gamma^1 = (\Omega A + \Omega \Delta B)\, \Gamma^0$$

One turn-map

$$\Gamma^N = (\Omega A + \Omega \Delta B)^N\, \Gamma^0$$

The group generated by $(\Omega, \Delta)$ is crucial to the understanding of the motion.

Theorem:

$\exists$ a basis

$$(\Omega A + \Omega \Delta B) = \begin{pmatrix} C_1 & & & \\ & \ddots & & \\ & & C_\mu & \\ & & & \ddots \\ & & & & C_N \end{pmatrix}$$

$$C_\mu = \omega_\mu A + \omega_\mu \delta_\mu B .$$

$$\omega_\mu = \begin{pmatrix} \cos\left(\frac{2\pi}{N}\mu\right) & \sin\left(\frac{2\pi}{N}\mu\right) \\ -\sin\left(\frac{2\pi}{N}\mu\right) & \cos\left(\frac{2\pi}{N}\mu\right) \end{pmatrix}$$

$$\delta_\mu = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} .$$

Derivation: Beyond my present mental abilities.

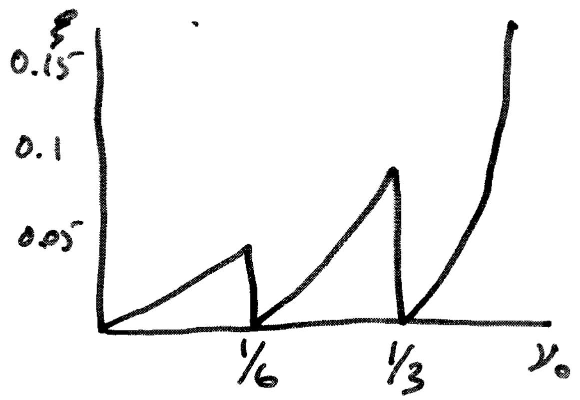
Head-on only:

$\varphi_1 = (x, x')$ $\quad \psi_1 = (y, y')$

4d problem / symplectic

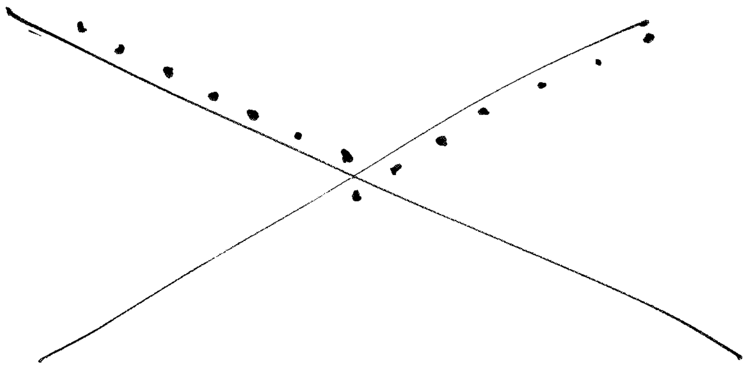
$\Downarrow$

exactly solvable






Long Range ?



First order operator at I.P.

$$H = \underbrace{\sum_{j=1}^{m} H_j}_{\text{Head on}} + \underbrace{\sum_{\substack{\ell \neq 0 \\ \ell = -J}}^{J} H_{j(j+\ell)}}_{\text{Long range}}$$

$$H_j = \frac{\epsilon}{2} \left( x_j - y_j \right)^2$$

$$H_{j(j+\ell)} = \frac{2\epsilon p}{L^2 \ell^2} \Big( \underbrace{\rho_\ell x_j}_{\text{phase advances}} - \underbrace{\rho_{-\ell} y_j}_{\text{retarded.}} \Big)^2$$

<u>Trick #2</u> Von Karman boundary condition.

B.S. <u>but who cares?</u>

Trick # 3

$$\begin{cases} \rho_\ell x = x + \frac{\ell L}{2} x' \\ \rho_\ell x' = x' \end{cases}$$

$$\Big\downarrow \text{ B.S.}$$

$$\begin{cases} \sigma_\ell x = \frac{\ell L}{2} x' \\ \sigma_\ell x' = \frac{-2}{\ell L} x^{\#} \end{cases}$$ "pseudo-drift".

Surprise $\Longrightarrow$ exactly solvable.

Fourier-Transforming Long-range.

$$\det \left( C_\mu^{\lambda} - \Lambda I \right) = 0$$