COHERENT BEAM-BEAM EFFECTS IN THE L.H.C.

SRINIVAS KRISHNAGOPAL

Centre for Advanced Technology, INDIA

Workshop on Beam-Beam Effect for Large Hadron Colliders
12-16, April 1999, CERN

OUTLINE

1. DETAILS OF THE CODE 'C.B.I.'
2. SOME GENERAL RESULTS ON COHERENT BBI
3. RESULTS FOR THE L.H.C.

Note that the focus here is on <u>quadrupole</u> coherent effects.

(I) THE CODE 'CBI'

- The code CBI (for Collective Beam-beam Interactions) is a self-consistent code that models the transverse beam-beam dynamics of beams of arbitrary distribution and ellipticity.

- It is a PIC code that calculates the beam-beam force on a 2-D Cartesian grid.

FEATURES :

→ Only one bunch per beam and only one collision point

→ Arc transport is linear

→ Radiation damping + fluctuations are put in once a turn and at one point in the ring

→ Transverse dimensions + distributions can be completely arbitrary

→ LONGITUDINAL DYNAMICS IS NOT MODELED

→ THERE IS NO CROSSING ANGLE

→ NO PARASITIC COLLISIONS

→ AN IDEAL TRANSVERSE FEEDBACK SYSTEM CAN BE SIMULATED

References :

(1) S. Krishnagopal, CAT Internal report 95-5 (1995)

(2) S. Krishnagopal, Phys. Rev. Lett., 76, 235 (1996)

$$(X_0, Y_0) \xrightarrow[\text{Transport}]{\text{Betatron}} (X', Y') \xrightarrow{\text{Radiation}} (X'', Y'') \xrightarrow{\text{BBI}} (X_1, Y_1)$$

<u>Betatron transport</u> :-

$$X' = M_x X_0$$
$$Y' = M_y Y_0$$

$$M_x = \begin{pmatrix} \cos 2\pi Q_x & \beta_x \sin 2\pi Q_x \\ -\frac{1}{\beta_x} \sin 2\pi Q_x & \cos 2\pi Q_x \end{pmatrix}$$

Similarly M_y.

<u>Radiation</u> :- Average effect, over one turn, is put in at one point in the ring.

$$X'' = M_r X' + X_f$$

$$M_r = \begin{pmatrix} e^{-\delta/2} & 0 \\ 0 & e^{-\delta/2} \end{pmatrix} \qquad \delta = \frac{U_0}{E_0}$$

$$X_f = \begin{pmatrix} \hat{r} \left[\beta_x \epsilon_{0x} (1 - e^{-\delta})\right]^{1/2} \\ \hat{r} \left[\frac{\epsilon_{0x}}{\beta_x} (1 - e^{-\delta})\right]^{1/2} \end{pmatrix}$$

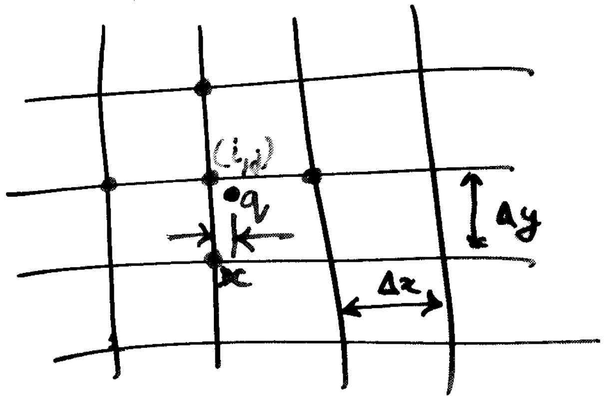
Beam-Beam Interaction:

CBI assumes: ultrarelativistic beams; single bunches; one collision per turn.

Uses a Cartesian, (x,y), grid.

① DENSITY CALCULATION:

Second-order weighting, assigning charge to the five nearest grid-points. (Quadratic spline)



$$\left.\begin{aligned} W(x_{i\pm 1}, y_j) &= \frac{1}{2}\left(\frac{1}{2} \pm \frac{x}{\Delta x}\right)^2 \\ W(x_i, y_j) &= \frac{3}{4} - \left(\frac{x}{\Delta x}\right)^2 \end{aligned}\right\} \quad 0 \le x < \frac{\Delta x}{2}$$

Similarly in y.

Use the FACR (Fourier Analysis and Cyclic Reduction) method of Hockney.

Solution to Poisson's eq. is obtained using a 5-point difference formula:

$$\nabla^2\phi_{i,j} = \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta x)^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta y)^2} = \frac{1}{\varepsilon_0}\rho_{i,j}$$

Three kinds of BCs allowed:

(a) Potential specified on boundaries;

(b) $\vec{\nabla}\phi$ is zero on boundaries;

(c) Potential is periodic.

Solution proceeds in the following 6 steps:

STEP 1. : Elimination of even lines.

Write above eqs. for all <u>rows</u> $i = 1, N_x$. Potential * $\phi_{i,j}$ can be written in terms of $\phi_{i\pm1,j}$, $\phi_{i\pm2,j}$ and $\phi_{i,j\pm2}$; i.e. on alternate lines.

Thus, reduce to $N_y/2$ coupled sets of N_x equations. Also get <u>modified charge</u> $q^*_{i,j}$, that depends on $q_{i,j}$, $q_{i\pm1,j}$, $q_{i,j\pm1}$.

STEP 2. :- Fourier analysis of odd lines.

Get $N_y/2$ independent sets of N_x equations, one for each of the N_x Fourier amplitudes.

The no. of eqs. is reduced by half in each cycle of recursion (as in step 1), until only one potential is left. Solve for it and reverse to get all the Fourier coeffs on odd lines.

EP 4.:- Fourier synthesis for potential on odd lines.

From Fourier coeffs. inverse FFT gives actual potentials on all the odd lines.

TEP 5.:- Modify charges on even lines

Setup recursive relations, and modify charges as in Step 1

TEP 6.:- Solution on even lines by cyclic reduction

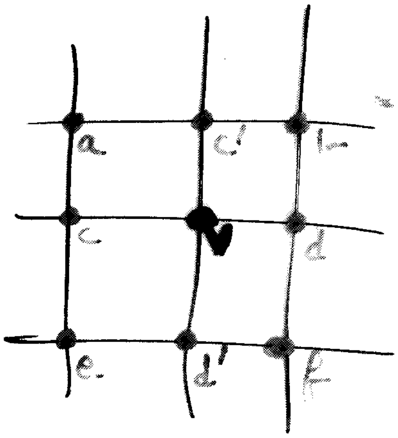
Proceed as in Step 3 to solve for potential on even lines by cyclic reduction.

) FIELD CALCULATION

Use a 6-point difference scheme:

$$E_x(v) = \frac{1}{3}\left[\frac{1}{2}\,\frac{\phi_b-\phi_a}{2(\Delta x)} + \frac{2(\phi_d-\phi_c)}{2(\Delta x)} + \frac{1}{2}\,\frac{\phi_f-\phi_e}{2(\Delta x)}\right]$$

$$E_y(v) = \frac{1}{3}\left[\frac{1}{2}\,\frac{\phi_a-\phi_e}{2(\Delta y)} + 2\,\frac{(\phi_{c'}-\phi_{d'})}{2(\Delta y)} + \frac{1}{2}\,\frac{\phi_b-\phi_f}{2(\Delta y)}\right]$$



FIELD INTERPOLATION :-

To conserve momentum, must use same order as density calculation, i.e. second-order weighting or quadratic spline

DIAGNOSTICS

① Start with random potential on grid. Differentiate twice to get density. Recalculate potential using FACR. This checks the Poisson solver.

② Initialize particles in a two-dimensional Gaussian, and use CBI to calculate the fields. Compare with analytic formula. Agrees well.

③ Check sensitivity to simulation parameters: no. of particles; grid spacing; grid size.

Use 10,000 test particles.
grid-spacing = 6/5
grid size = 256

④ Ran ROUND beams, and compared with results of earlier paper. They agree.

⑤ Built-in diagnostic :- Every 100 turns, X & Y slices of the potential are taken and differentiated to recalculate the charge density. Require that the recalculated and original densities agree at every grid-point to within a specified limit (presently 10%).

→ It is generally agreed that COHERENT beam-beam effects play an important role in luminosity limitation.

BUT THE DYNAMICAL MECHANISM IS NOT KNOWN

Some Experimental Observations:

(a) Measurements of steady-state beam distributions show large non-Gaussian tails. [$>3\sigma$]

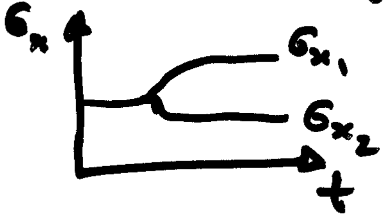
(b) Some measurements also show evidence of non-Gaussian character in the core of the beam [$<2\sigma$]

[Measurements at CESR; GPJ Thesis]

(c) One effect, widely observed in many colliders, is a [redacted] the so-called FLIP-FLOP EFFECT. Here one beam is blown up to a very large size while the other becomes small.

→ Focus

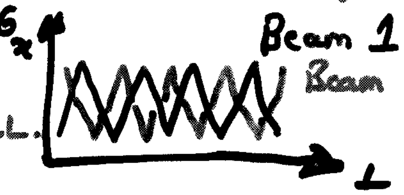
⇒ Overlap is poor ⇒ luminosity is limited



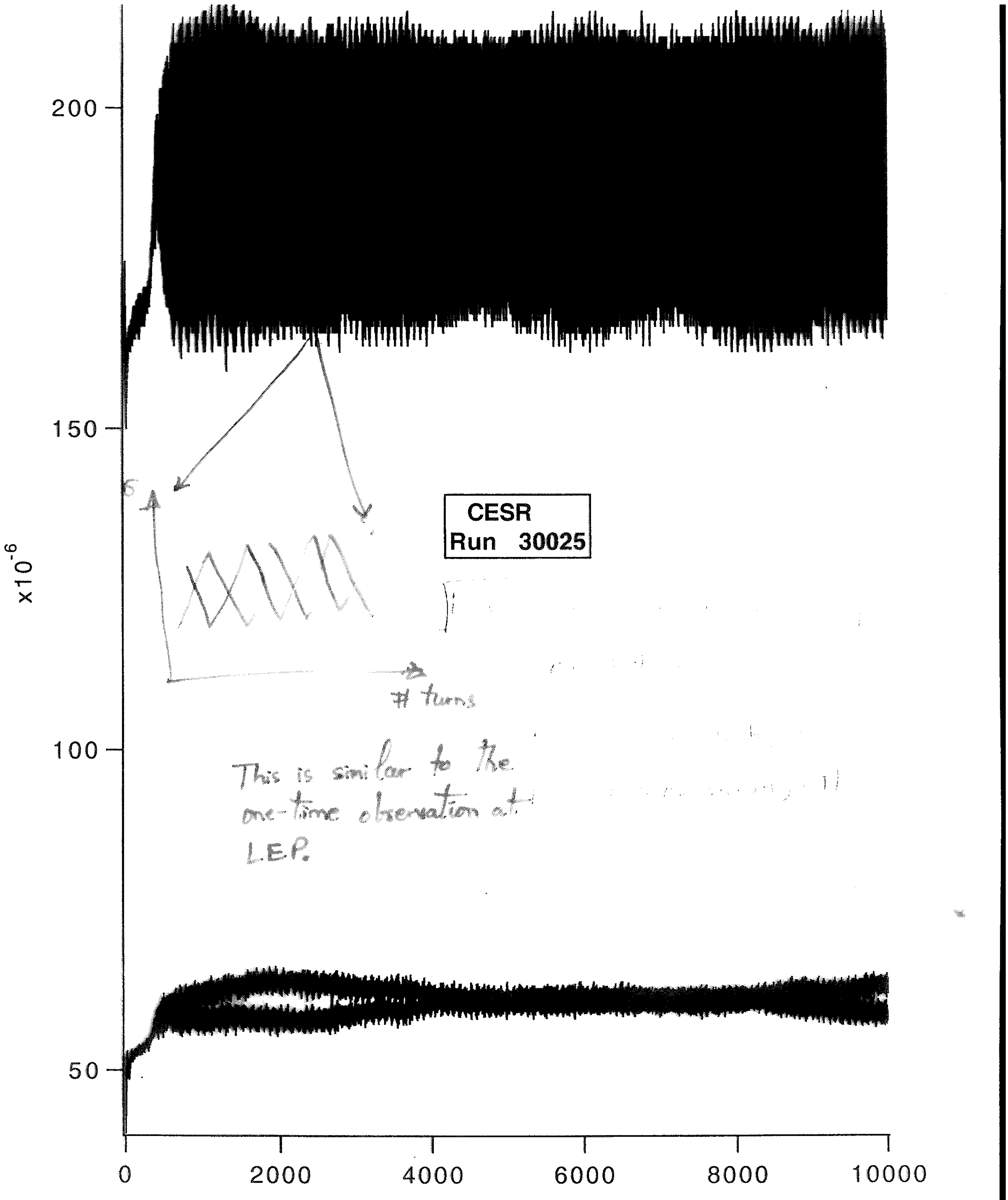
(d) In a second effect, recently observed at LEP in CERN, it was found that the beam sizes vary rapidly, turn-by-turn, in a periodic manner.

→ Focus

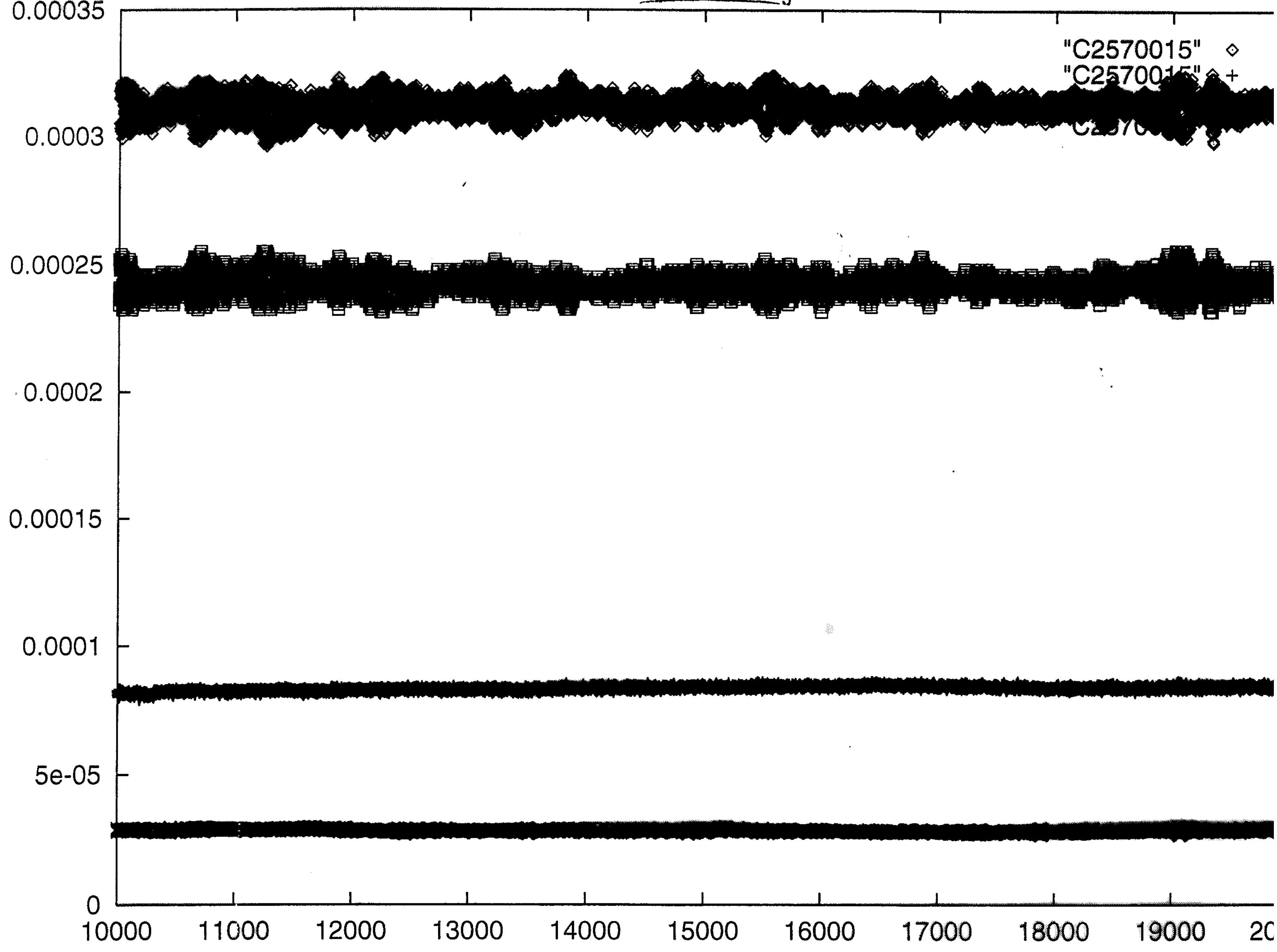
PERIOD-n OSCILL.



Again, poor overlap ⇒ lum. limi


CESR
Run 30025
x10^-6
200
150
100
50
0
2000
4000
6000
8000
10000
turns
This is similar to the
one-time observation at
L.E.P.

40 mA



0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
0
10000
11000
12000
13000
14000
15000
16000
17000
18000
19000
"C2570015"

SIMULATIONS FOR THE L.HC.

Issues :-
- Are there quadrupole coherent instabilities for nominal LHC parameters?
- What is the effect of sweeping one beam around the other?

LHC Parameters :-

$E_0 = 7$ TeV

$\epsilon_x = \epsilon_y = 5 \times 10^{-10}$ m-rad

$\beta_x = \beta_y = 0.5$m

$I_{bunch} = 0.2$mA $(\Rightarrow N = \sim 1.5 \times 10^{11})$

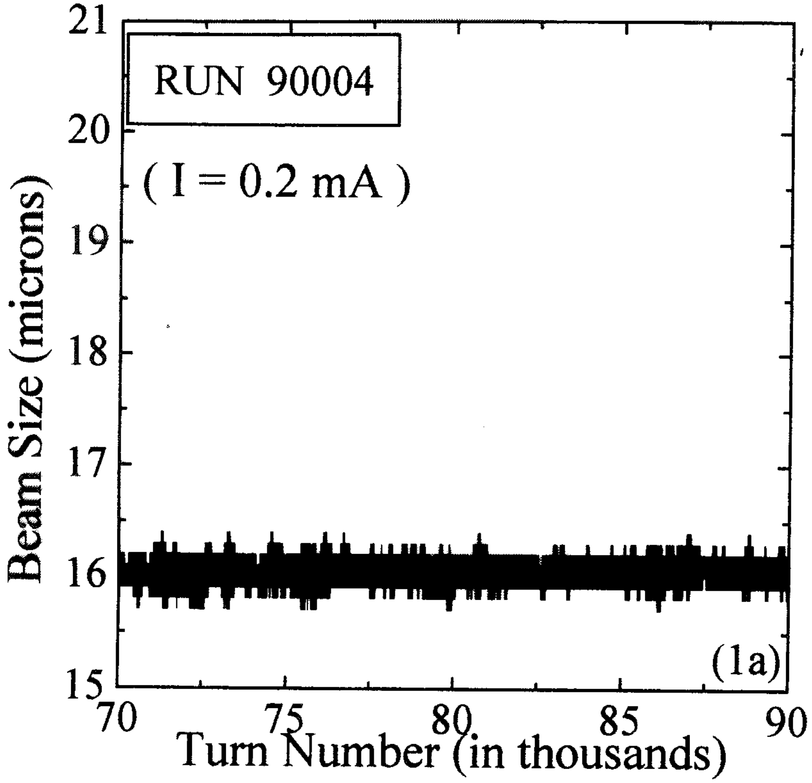
$Q_H = 0.28$

$Q_V = 0.31$

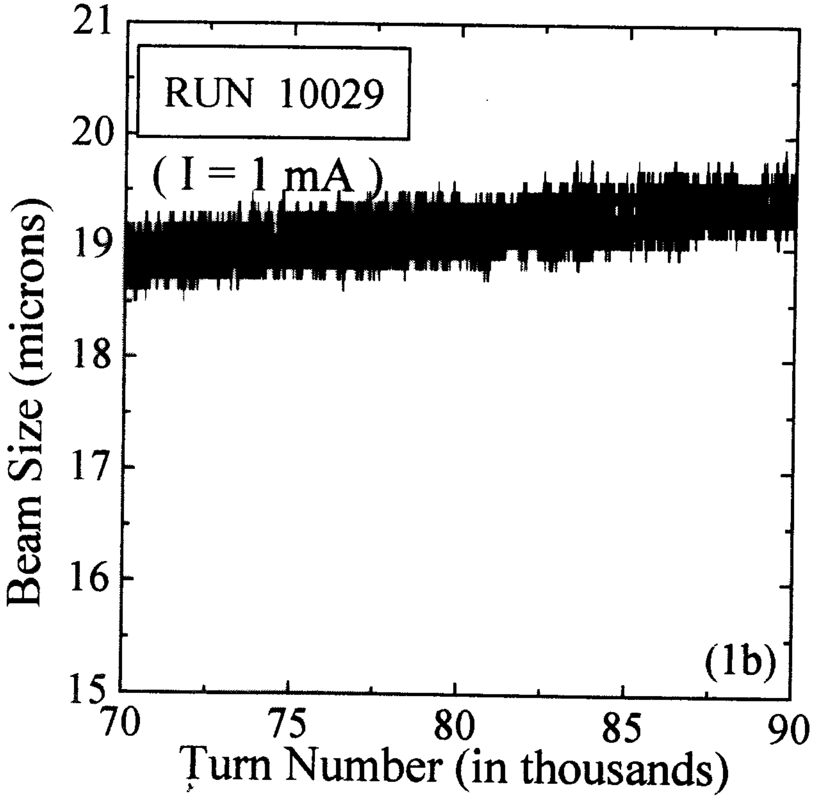
No RADIATION DAMPING

[Head-on collisions]

(a) I = 0·2mA



(b) I = 1 mA

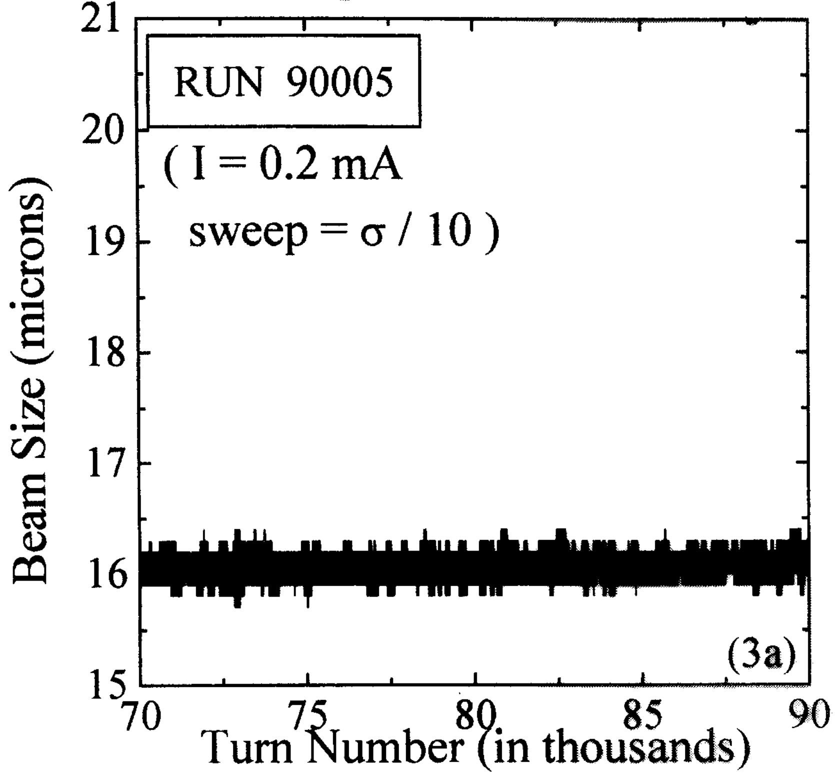


Beam 1 being swept around beam 2

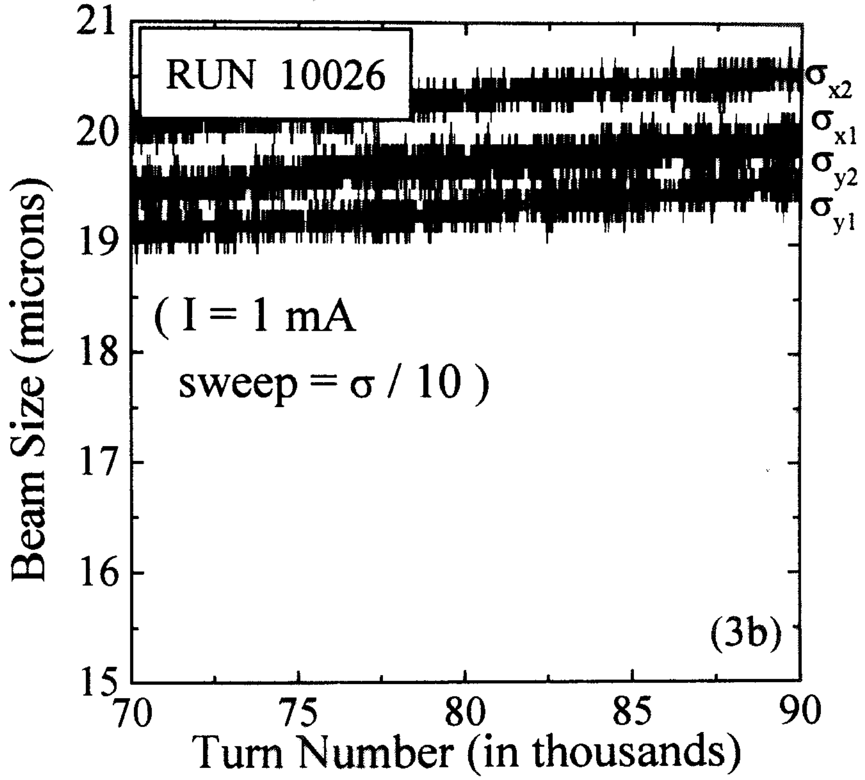
Period = 10 turns

Disp = 6/10

(a) I = 0.2mA



(b) I = 1 mA

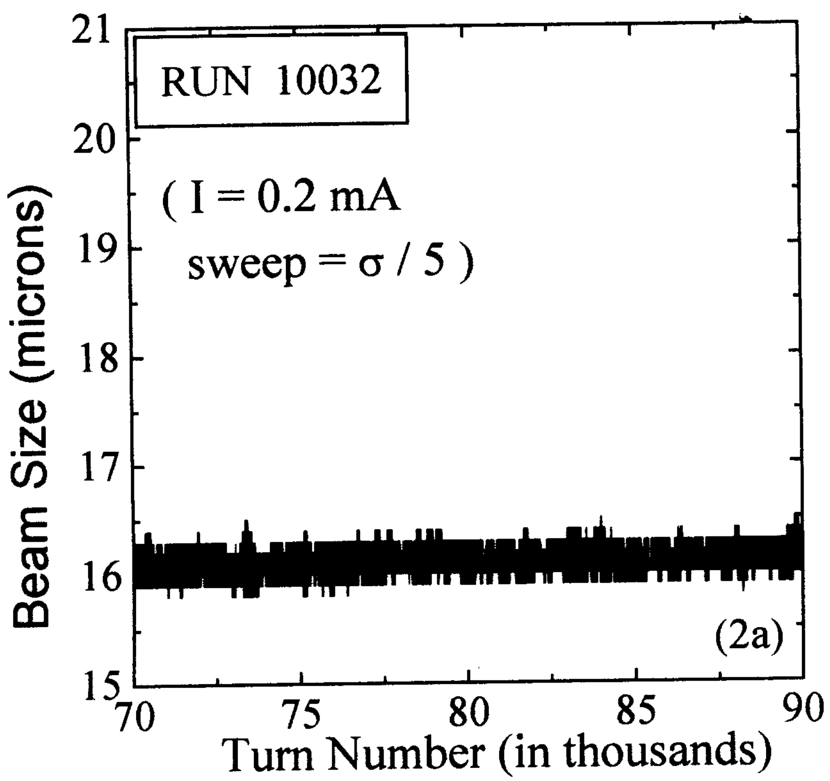


Beam 1 being swept around beam 2

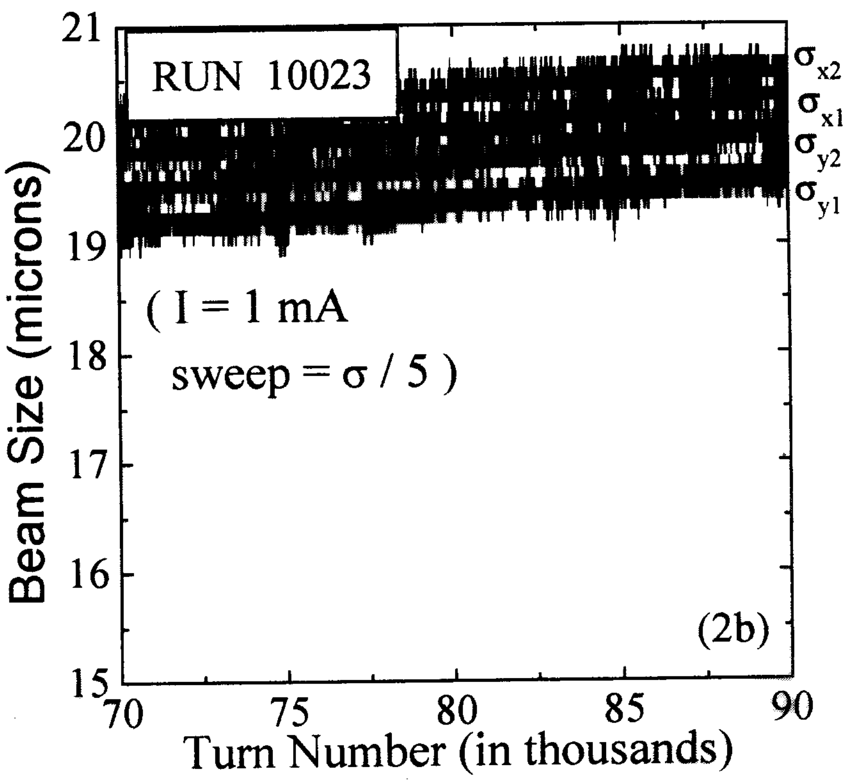
Period = 10 turns

Disp = 6/5

(a) I = 0.2 mA



(b) I = 1 mA



turn # 1 2 3

Current (mA)	σ_{x_1} (μm)	σ_{y_1} (μm)	σ_{x_2} (μm)	σ_{y_2} (μm)
0.1	15.8	15.8	15.8	15.8
0.2	16.2	16.1	16.1	16.1
0.5	17.4	17.4	17.1	16.9
1.0	20.6	20.0	20.4	19.6
2.0	28.1	25.4	27.6	25.1

Table: All beam sizes after 90,000 turns, at different currents, for displ. = $\sigma/5$.

Summary of observations for LHC

① At nominal parameters, there is no coherent behaviour

② Sweeping one beam around the other **does** induce unequal beam sizes

③ This effect is noticable only around $I = 0.5\,mA$ [redacted], well above the nominal current (0.2 mA)

Future work :-

① Influence of longitudinal dynamics

② Influence of crossing-angle

③ Influence of parasitic collisions

* ④ Effect on **dipole** oscillations

⑤ Repeat [redacted] earlier work, without feedback